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**STRATEGIC PROFIT SHARING BETWEEN FIRMS:
AN APPLICATION TO JOINT VENTURES ***

*Roberts Waddle*¹

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Our companion article developed a clear conceptual framework of profit sharing between two rival firms and studied the effects of this strategy on each firm's profit under the assumption that each firm decides unilaterally to give away voluntarily a part of its profit to its rival. This article relaxes totally this assumption and allows firms to invest rather a fraction of their profits in a joint venture.

As in the previous article, it shows *how and when forming a joint venture may be a successful strategy*. Furthermore and more importantly, it brings to light that joint venture may be used to *conceal the profit-sharing (maybe forbidden) strategy*.

Key Words: Profit sharing, Oligopoly, Competition, Joint venture.

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Strategic Profit Sharing Between Firms: An Application to Joint Ventures¹

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Preliminary- Comments welcome!
(*please, do not circulate*)

¹I'm very grateful to my supervisor José Luis Ferreira for his numerous helpful suggestions. Nevertheless, all remained errors are my own.

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Abstract

Our companion article developed a clear conceptual framework of profit sharing between two rival firms and studied the effects of this strategy on each firm's profit under the assumption that *each firm decides unilaterally to give away voluntarily a part of its profit to its rival*. This article relaxes totally this assumption and allows firms to *invest rather* a fraction of their profits in *a joint venture*.

It thus shows how and when forming a joint venture may be a successful strategy. Furthermore and more importantly, it brings to light that *a joint venture may be used to conceal the profit-sharing (maybe forbidden) strategy*.

Keywords: Profit sharing, Oligopoly, Competition, Joint venture.

JEL Classification: C72, D21, L13, L24.

1 Introduction

In a companion paper (Waddle 2005b), we examined how sharing profits may increase firms' profits in an oligopoly market.

Our companion paper¹ focused on such a strategy where both firms unilaterally decides to *give away* a fraction of their profits to their rivals. The purpose of this present paper is to relax totally this assumption and to allow firms to *invest rather* a part of their profits in a joint venture (henceforth JV). The rationale of relaxing that hypothesis is to overcome any eventual prohibition of this strategy.

It thus shed light on how and when forming a JV may be a successful strategy. Furthermore and more importantly, it brings to light that *a JV may be used to conceal the profit-sharing (maybe forbidden) strategy*. Interestingly, this result holds for differentiated markets too.

The article proceeds as follows. Section 2 presents the model where firms receive half the JV profit to show the existence of a multiplicity of NE_a in prices. It then points out that firms win by investing in a JV. Section 3 modifies the model by allowing firms to share the JV profit according to a "proportion rule". However, the result remains the same. As before, it highlights that invest in a JV is a winning strategy. Section 4 concludes.

2 The model

We consider here a model similar to the one presented in our companion paper (Waddle 2005b) except that we allow firms to *invest* (rather than *to share*) a part of their profits to a joint venture.

Let two firms 1 and 2 in a homogeneous market and suppose that each firm incurs a cost c per unit of production. The market demand function is $q = D(p) = 1 - p$. We assume that firms do not have capacity constraints and always supply the demand they face. Therefore, the profit function of firm i is:

$$\Pi_i = \begin{cases} (p_i - c)q_i & \text{if } p_i < p_j \\ \frac{1}{2}(p_i - c)q_i & \text{if } p_i = p_j \\ 0 & \text{otherwise} \end{cases} \quad i = 1, 2 \ (i \neq j)$$

where q_i is the quantity demanded faced by firm i .

¹See Waddle 2005b for a discussion of the relation between our work and the literature.

Let α_1 (resp. α_2) denote the part of the profit that firm 1 (resp. firm 2) is willing to invest in the JV. We assume that $\alpha_i \in [0, 1]$. Since $\alpha_1\Pi_1 + \alpha_2\Pi_2$ is invested to the JV, we can suppose that the JV profit function is: $f(\alpha_1\Pi_1, \alpha_2\Pi_2) = \gamma(\alpha_1\Pi_1 + \alpha_2\Pi_2)$ where $\gamma > 0$ is the technology of the JV production². Finally, we assume that each firm receives half the JV profit³.

Consequently, we can write the *new profit function* $P_i(p_i(\alpha_i, \alpha_j), p_j(\alpha_i, \alpha_j))$ (hereafter P_i) of each firm as:

$$P_i = (1 - \alpha_i)\Pi_i + \frac{1}{2}f(\alpha_1\Pi_1, \alpha_2\Pi_2) = (1 - \alpha_i)\Pi_i + \frac{1}{2}\gamma(\alpha_i\Pi_i + \alpha_j\Pi_j) \text{ or}$$

$$P_i = \left[1 - \left(1 - \frac{1}{2}\gamma\right)\alpha_i\right] \Pi_i + \frac{1}{2}\gamma\alpha_j\Pi_j$$

We consider a two-stage game whose sequences are thus defined. In the first stage of the game, firms choose the optimal α_i to invest. In the second stage of the game, firms select p_i .

In the *first stage of the game*, for α_1 and α_2 firms simultaneously solve:

$$Max_{\alpha_1} \quad P_1 = \left[1 - \left(1 - \frac{1}{2}\gamma\right)\alpha_1\right] \Pi_1 + \frac{1}{2}\gamma\alpha_2\Pi_2$$

$$Max_{\alpha_2} \quad P_2 = \left[1 - \left(1 - \frac{1}{2}\gamma\right)\alpha_2\right] \Pi_2 + \frac{1}{2}\gamma\alpha_1\Pi_1$$

In the *second stage of game*, for p_1 and p_2 firms simultaneously solve:

$$Max_{p_1} \quad P_1 = \left[1 - \left(1 - \frac{1}{2}\gamma\right)\alpha_1\right] \Pi_1 + \frac{1}{2}\gamma\alpha_2\Pi_2$$

$$Max_{p_2} \quad P_2 = \left[1 - \left(1 - \frac{1}{2}\gamma\right)\alpha_2\right] \Pi_2 + \frac{1}{2}\gamma\alpha_1\Pi_1$$

²Note that the joint venture profit function, by analogy to the traditional macro production function, can be decreasing, constant or increasing return to scale whether γ is less, equal or greater than 1.

³In the next section, we will relax this assumption by giving out the JV profit following a "proportion rule": $\frac{\alpha_i}{\alpha_i + \alpha_j}$

2.1 Solving the second-stage of the game

To find the subgame perfect Nash equilibrium (SPNE), we begin by solving subgames in the second-stage. Recall that, in the second stage, firms are looking for prices that maximize their profits.

Proposition 1 *If $(1 - \frac{1}{2}\gamma)\alpha_1 + \frac{1}{2}\gamma\alpha_2 = 1$ and $(1 - \frac{1}{2}\gamma)\alpha_2 + \frac{1}{2}\gamma\alpha_1 = 1$, then any prices (p_1, p_2) such that $c \leq p_1 = p_2 \leq p_m$ are NE_a in the second stage of the game*

Proof. (p_1, p_2) such that $c \leq p_1 = p_2 \leq p_m$ are NE_a if and only if no firm wants to deviate from those prices by fixing a price p'_i above or below. In fact:

$$c \leq p_1 = p_2 = p \leq p_m \Rightarrow \Pi_1 = \Pi_2 \geq 0$$

$$\Pi_1 = \frac{1}{2} (p_1 - c) (1 - p_1) = \frac{1}{2} (p - c) (1 - p) = \Pi_2$$

$$P_1 = \left[1 - (1 - \frac{1}{2}\gamma)\alpha_1 + \frac{1}{2}\gamma\alpha_2\right] \Pi_1$$

$$P_1 = \frac{1}{2} \left[1 - (1 - \frac{1}{2}\gamma)\alpha_1 + \frac{1}{2}\gamma\alpha_2\right] (p - c) (1 - p)$$

$$P_2 = \frac{1}{2} \left[1 - (1 - \frac{1}{2}\gamma)\alpha_2 + \frac{1}{2}\gamma\alpha_1\right] (p - c) (1 - p)$$

Suppose that:

$$i) p_1 = p_2 - \varepsilon \ (\varepsilon > 0) \iff \Pi_1 = (1 - p_1) (p_1 - c) \text{ and } \Pi_2 = 0$$

$$P'_1 = \left[1 - (1 - \frac{1}{2}\gamma)\alpha_1\right] \Pi_1 = \left[1 - (1 - \frac{1}{2}\gamma)\alpha_1\right] (1 - p_1) (p_1 - c)$$

If $p_1 \leq p_m$ (monopolistic price), then $p_1 = p - \varepsilon$.

$$\text{For } \varepsilon \text{ very small}^4, P'_1 \simeq \left[1 - (1 - \frac{1}{2}\gamma)\alpha_1\right] (1 - p) (p - c) \leq P_1 \iff$$

$$\left[1 - (1 - \frac{1}{2}\gamma)\alpha_1\right] \leq \frac{1}{2} \left[1 - (1 - \frac{1}{2}\gamma)\alpha_1 + \frac{1}{2}\gamma\alpha_2\right] \quad (1)$$

$$ii) p_1 = p_2 + \varepsilon \ (\varepsilon > 0) \iff \Pi_2 = (1 - p_2) (p_2 - c) > 0 \text{ and } \Pi_1 = 0$$

⁴There is no reason for not to suppose that ε is very small. For instance, firms need to decrease or increase just slightly to get or to lose the entire market.

$$P_1'' = \frac{1}{2}\gamma\alpha_2\Pi_2 = \frac{1}{2}\gamma\alpha_2(1-p_2)(p_2-c) \leq P_1 \Leftrightarrow$$

$$\frac{1}{2}\gamma\alpha_2 \leq \frac{1}{2} \left[1 - (1 - \frac{1}{2}\gamma)\alpha_2 + \frac{1}{2}\gamma\alpha_1 \right] \quad (2)$$

Equations (1) and (2) represent the non-deviation conditions for firms 1 and are both satisfied when $[1 - (1 - \frac{1}{2}\gamma)\alpha_1] = \frac{1}{2}\gamma\alpha_2$ or $(1 - \frac{1}{2}\gamma)\alpha_1 + \frac{1}{2}\gamma\alpha_2 = 1$.

Likewise, we can show that firm 2 will not deviate if $[1 - (1 - \frac{1}{2}\gamma)\alpha_2] = \frac{1}{2}\gamma\alpha_1$ or $(1 - \frac{1}{2}\gamma)\alpha_2 + \frac{1}{2}\gamma\alpha_1 = 1$.

Conclusion: if $(1 - \frac{1}{2}\gamma)\alpha_1 + \frac{1}{2}\gamma\alpha_2 = 1$ & $(1 - \frac{1}{2}\gamma)\alpha_2 + \frac{1}{2}\gamma\alpha_1 = 1$, then any prices (p_1, p_2) such that $c \leq p_1 = p_2 \leq p_m$ are NE_a in the second-stage of the game. ■

Proposition 2 *If $(1 - \frac{1}{2}\gamma)\alpha_1 + \frac{1}{2}\gamma\alpha_2 = 1$ or $(1 - \frac{1}{2}\gamma)\alpha_2 + \frac{1}{2}\gamma\alpha_1 = 1$, then any prices (p_i, p_j) such that $c \leq p_i = p_m < p_j$ are NE_a in the second stage of the game*

Proof. (p_1, p_2) s. t. $c \leq p_2 = p_m < p_1$ are NE_a if and only if no firm has interest to deviate from those prices by fixing a price p'_i above or below.

$$A.- c \leq p_2 = p_m < p_1 \Rightarrow \Pi_1 = 0 \text{ and } \Pi_2 = (p_2 - c)(1 - p_2) > 0$$

$$P_1 = \frac{1}{2}\gamma\alpha_2\Pi_2 = \frac{1}{2}\gamma\alpha_2(p_2 - c)(1 - p_2)$$

$$P_2 = [1 - (1 - \frac{1}{2}\gamma)\alpha_2] \Pi_2 = [1 - (1 - \frac{1}{2}\gamma)\alpha_2] (p_2 - c)(1 - p_2)$$

Suppose that:

$$i) p_1 = p_2 - \varepsilon \ (\varepsilon > 0) \Leftrightarrow \Pi_1 = (1 - p_1)(p_1 - c) \text{ and } \Pi_2 = 0$$

$$P'_1 = [1 - (1 - \frac{1}{2}\gamma)\alpha_1] \Pi_1 = [1 - (1 - \frac{1}{2}\gamma)\alpha_1] (1 - p_2 + \varepsilon)(p_2 - \varepsilon - c)$$

$$\text{For } \varepsilon \text{ very small, } P'_1 \simeq [1 - (1 - \frac{1}{2}\gamma)\alpha_1] (1 - p_2)(p_2 - c) < P_1 \Leftrightarrow$$

$$\left[1 - (1 - \frac{1}{2}\gamma)\alpha_1 \right] < \frac{1}{2}\gamma\alpha_2 \quad (3)$$

Equation (3) represent the non-deviation conditions for firm 1 and can be rewritten as: $(1 - \frac{1}{2}\gamma)\alpha_1 + \frac{1}{2}\gamma\alpha_2 > 1$

$$B.- c \leq p_1 = p_m < p_2 \Rightarrow \Pi_1 = (p_1 - c)(1 - p_1) \text{ and } \Pi_2 = 0$$

$$P_1 = [1 - (1 - \frac{1}{2}\gamma)\alpha_1] \Pi_1 = [1 - (1 - \frac{1}{2}\gamma)\alpha_1] (p_1 - c)(1 - p_1)$$

$$P_2 = \frac{1}{2}\gamma\alpha_1\Pi_1 = \frac{1}{2}\gamma\alpha_1 (p_1 - c)(1 - p_1)$$

Suppose that:

$$i) p_2 = p_1 - \varepsilon \ (\varepsilon > 0) \iff \Pi_2 = (1 - p_2)(p_2 - c) \text{ and } \Pi_2 = 0$$

$$P'_2 = [1 - (1 - \frac{1}{2}\gamma)\alpha_2] \Pi_2 = [1 - (1 - \frac{1}{2}\gamma)\alpha_2] (1 - p_1 + \varepsilon)(p_1 - \varepsilon - c)$$

$$\text{For } \varepsilon \text{ very small, } P'_1 \simeq [1 - (1 - \frac{1}{2}\gamma)\alpha_2] (1 - p_1)(p_1 - c) < P_1 \Leftrightarrow$$

$$\left[1 - (1 - \frac{1}{2}\gamma)\alpha_2\right] < \frac{1}{2}\gamma\alpha_1 \quad (4)$$

Equation (4) represent the non-deviation conditions for firm 2 and can be rewritten as: $(1 - \frac{1}{2}\gamma)\alpha_2 + \frac{1}{2}\gamma\alpha_1 > 1$

Conclusion: if $(1 - \frac{1}{2}\gamma)\alpha_2 + \frac{1}{2}\gamma\alpha_1 > 1$ or $(1 - \frac{1}{2}\gamma)\alpha_1 + \frac{1}{2}\gamma\alpha_2 > 1$, then any prices (p_i, p_j) such that $c \leq p_i = p_m < p_j$ constitute NE_a in the second-stage of the game. ■

Proposition 3 *If $(1 - \frac{1}{2}\gamma)\alpha_2 + \frac{1}{2}\gamma\alpha_1 < 1$ or $(1 - \frac{1}{2}\gamma)\alpha_1 + \frac{1}{2}\gamma\alpha_2 < 1$, then any price (p_1, p_2) such that $p_1 = p_2 = c$ is NE_a in the second stage of the game*

Proof. (p_1, p_2) s.t. $p_1 = p_2 = c$ is NE if and only if no firm has interest to deviate from those prices to fix a price p'_i above or below.

$$p_2 = p_2 = c \Rightarrow \Pi_1 = 0 \text{ and } \Pi_2 = 0$$

$$P_1 = [1 - (1 - \frac{1}{2}\gamma)\alpha_1] \Pi_1 + \frac{1}{2}\gamma\alpha_2\Pi_2 = 0$$

$$P_2 = [1 - (1 - \frac{1}{2}\gamma)\alpha_2] \Pi_2 + \frac{1}{2}\gamma\alpha_1\Pi_1 = 0$$

We shall study separately the deviation for both firms. Let us check first for firm 1. Suppose that:

$$i) p_1 = p_2 - \varepsilon \ (p_1 < p_2 \text{ and } p_1 < c) \Rightarrow \Pi_1 < 0 \text{ and } \Pi_2 = 0$$

$$P'_1 = [1 - (1 - \frac{1}{2}\gamma)\alpha_1] \Pi_1 < 0$$

$\Rightarrow P'_1 < P_1 = 0 \Rightarrow$ Firm 1 has no interest by fixing a price below p_2

ii) $p_1 = p_2 + \varepsilon$ ($p_1 > p_2 = c$) $\iff \Pi_2 = (1 - p_2)(p_2 - c) = 0$ and $\Pi_1 = 0$ (firm 1 does not produce)

$$P''_1 = \frac{1}{2}\gamma\alpha_2\Pi_2 = 0$$

$P''_1 = 0 = P_1 \Rightarrow$ Firm 1 has no interest by fixing a price above p_2

Let us check now for firm 2. Suppose that:

i) $p_2 = p_1 - \varepsilon$ ($p_2 < p_1$ and $p_2 < c$) $\Rightarrow \Pi_2 < 0$ and $\Pi_1 = 0$ (firm 1 does not produce)

$$P'_2 = [1 - (1 - \frac{1}{2}\gamma)\alpha_2] \Pi_2 < 0$$

$\Rightarrow P'_2 < P_2 = 0 \Rightarrow$ Firm 2 has no interest by fixing a price below p_1

ii) $p_2 = p_1 + \varepsilon$ ($p_2 > p_1 = c$) $\iff \Pi_1 = (1 - p_1)(p_1 - c) = 0$ and $\Pi_2 = 0$ (firm 2 does not produce)

$$P''_2 = \frac{1}{2}\gamma\alpha_1\Pi_1 = 0$$

$P''_2 = 0 = P_2 \Rightarrow$ Firm 2 has no interest by fixing a price above p_1

Conclusion: If $(1 - \frac{1}{2}\gamma)\alpha_2 + \frac{1}{2}\gamma\alpha_1 < 1$ or $(1 - \frac{1}{2}\gamma)\alpha_1 + \frac{1}{2}\gamma\alpha_2 < 1$, then any price, (p_1, p_2) s.t. $p_1 = p_2 = c$ is a NE in the second-stage of the game. ■

The second-stage being entirely solved and NE_a being found, we can thus move to the first-stage of the game in order to find $SPNE_a$

2.2 Solving the first-stage of the game

In the first-stage of the game, firms choose the α_i optimal maximizing their profit to share with their rival.

Solving backwards, we have solved the second-stage of the game in the previous section and have found NE_a in prices summarized below⁵:

i) $(p_1, p_2) / p_1 = p_2 = c$ if $(1 - \frac{1}{2}\gamma)\alpha_2 + \frac{1}{2}\gamma\alpha_1 < 1$ or $(1 - \frac{1}{2}\gamma)\alpha_1 + \frac{1}{2}\gamma\alpha_2 < 1$ with:

⁵One can easily check that: if $(1 - \frac{1}{2}\gamma)\alpha_2 + \frac{1}{2}\gamma\alpha_1 = 1$ (resp $(1 - \frac{1}{2}\gamma)\alpha_1 + \frac{1}{2}\gamma\alpha_2 = 1$) then $\frac{1}{2} [1 - (1 - \frac{1}{2}\gamma)\alpha_1 + \frac{1}{2}\gamma\alpha_2] = 3\gamma\alpha_2$ (resp. $\frac{1}{2} [1 - (1 - \frac{1}{2}\gamma)\alpha_2 + \frac{1}{2}\gamma\alpha_1] = 3\gamma\alpha_1$).

$$\begin{cases} P_1 = 0 \\ P_2 = 0 \end{cases}$$

ii) $(p_1, p_2) / c \leq p_1 = p_2 \leq p_m$ if $(1 - \frac{1}{2}\gamma)\alpha_2 + \frac{1}{2}\gamma\alpha_1 = 1$ & $(1 - \frac{1}{2}\gamma)\alpha_1 + \frac{1}{2}\gamma\alpha_2 = 1$ with:

$$\begin{cases} P_1 = 3\gamma\alpha_2 (p - c) (1 - p) \\ P_2 = 3\gamma\alpha_1 (p - c) (1 - p) \end{cases}$$

iii) $(p_1, p_2) / c \leq p_i = p_m < p_j$ if $(1 - \frac{1}{2}\gamma)\alpha_2 + \frac{1}{2}\gamma\alpha_1 > 1$ or $(1 - \frac{1}{2}\gamma)\alpha_1 + \frac{1}{2}\gamma\alpha_2 > 1$ with:

$$\begin{cases} P_j = \frac{1}{2}\gamma\alpha_i (p_m - c) (1 - p_m) \\ P_i = [1 - (1 - \frac{1}{2}\gamma)\alpha_i] (p_m - c) (1 - p_m) \end{cases}$$

Now, in the current section, we draw our attention to the first-stage of the game searching for $SPNE_a$ in α_i .

Proposition 4 *The strategies $(\alpha_1, p_1(\alpha_1, \alpha_2))$, $(\alpha_1, p_1(\alpha_1, \alpha_2))$ s.t.:*

- i) $\alpha_i \in]0, 1[$ & $(1 - \frac{1}{2}\gamma)\alpha_2 + \frac{1}{2}\gamma\alpha_1 = 1$ & $(1 - \frac{1}{2}\gamma)\alpha_1 + \frac{1}{2}\gamma\alpha_2 = 1$
- ii) $\begin{cases} p_1^* = p_2^* = c \text{ if } (1 - \frac{1}{2}\gamma)\alpha_2 + \frac{1}{2}\gamma\alpha_1 < 1 \text{ or } (1 - \frac{1}{2}\gamma)\alpha_1 + \frac{1}{2}\gamma\alpha_2 < 1 \\ p_1^* = p_2^* = p_m \text{ if } (1 - \frac{1}{2}\gamma)\alpha_2 + \frac{1}{2}\gamma\alpha_1 = 1 \text{ and } (1 - \frac{1}{2}\gamma)\alpha_1 + \frac{1}{2}\gamma\alpha_2 = 1 \\ c \leq p_i = p_m < p_j \text{ if } (1 - \frac{1}{2}\gamma)\alpha_2 + \frac{1}{2}\gamma\alpha_1 > 1 \text{ or } (1 - \frac{1}{2}\gamma)\alpha_1 + \frac{1}{2}\gamma\alpha_2 > 1 \end{cases}$
are $SPNE_a$ of the game

Proof. The strategies $(\alpha_1, p_1(\alpha_1, \alpha_2))$, $(\alpha_1, p_1(\alpha_1, \alpha_2))$ s.t. i) and ii) are satisfied, are $SPNE_a$ if and only if no firm has interest to deviate from those prices by choosing a α'_i above or below. Because of the multiplicity of α_i , we investigate separately the deviation for each firm.

Let us check first for firm 1. Suppose that:

$$i) \alpha'_1 < \alpha_1 \Rightarrow (1 - \frac{1}{2}\gamma)\alpha'_1 + \frac{1}{2}\gamma\alpha_2 < 1 \Rightarrow$$

$$P'_1 = 0 < P_1 = 3\gamma\alpha_2 (p_m - c) (1 - p_m) \quad (5)$$

$$ii) \alpha'_1 > \alpha_1 \Rightarrow (1 - \frac{1}{2}\gamma)\alpha'_1 + \frac{1}{2}\gamma\alpha_2 > 1 \Rightarrow$$

$$P''_1 = \frac{1}{2}\gamma\alpha_2 (p_m - c) (1 - p_m) < P_1 = 3\gamma\alpha_2 (p_m - c) (1 - p_m) \quad (6)$$

(5) and (6) show that firm 1 has no interest to deviate.

Now, let us check for firm 2. Suppose that:

$$i) \alpha'_2 < \alpha_2 \Rightarrow (1 - \frac{1}{2}\gamma)\alpha'_2 + \frac{1}{2}\gamma\alpha_1 < 1 \Rightarrow$$

$$P'_2 = 0 < P_2 = 3\gamma\alpha_1(p_m - c)(1 - p_m) \quad (7)$$

$$ii) \alpha'_2 > \alpha_2 \Rightarrow (1 - \frac{1}{2}\gamma)\alpha'_2 + \frac{1}{2}\gamma\alpha_1 > 1 \Rightarrow$$

$$P'_2 = \left[1 - \left(1 - \frac{1}{2}\gamma\right)\alpha'_2\right](p_m - c)(1 - p_m) < P_2 = 3\gamma\alpha_1(p_m - c)(1 - p_m) \quad (8)$$

(7) and (8) show that firm 2 has no interest to deviate.

Conclusion: The strategies $(\alpha_1, p_1(\alpha_1, \alpha_2))$, $(\alpha_1, p_1(\alpha_1, \alpha_2))$ s.t. *i*) and *ii*) are satisfied, are SPNE_a of the game. ■

3 The modified model

We consider the same model as in the previous section except that we allow firms to share the JV profit following a "proportion rule".

As before, let two firms 1 and 2 in a homogeneous market and suppose that each firm incurs a cost c per unit of production. The market demand function is $q = D(p) = 1 - p$. We still assume that firms do not have capacity constraints and always supply the demand they face. Therefore, the profit function of firm i is:

$$\Pi_i = \begin{cases} (p_i - c)q_i & \text{if } p_i < p_j \\ \frac{1}{2}(p_i - c)q_i & \text{if } p_i = p_j \\ 0 & \text{otherwise} \end{cases} \quad i = 1, 2 \ (i \neq j)$$

where q_i is the quantity demanded faced by firm i .

Let α_1 (resp. α_2) denote the part of the profit that firm 1 (resp. firm 2) is willing to invest to a JV. We assume that $\alpha_i \in [0, 1]$. Since $\alpha_1\Pi_1 + \alpha_2\Pi_2$ is invested to the JV, we can suppose that the JV profit function is: $f(\alpha_1\Pi_1, \alpha_2\Pi_2) = \gamma(\alpha_1\Pi_1 + \alpha_2\Pi_2)$ where $\gamma > 0$ is the technology of the JV production⁶ Finally, contrary to the last section, we assume that firm

⁶Note that the joint venture profit function, by analogy to the traditional macro production function, can be decreasing, constant or increasing return to scale whether γ is less, equal or greater than 1.

i receives $\frac{\alpha_i}{\alpha_i + \alpha_j}$ of the JV profit. For simplicity, let us denote $C_i = \frac{\alpha_i}{\alpha_i + \alpha_j}$, $i = 1, 2$ ($i \neq j$)

Consequently, we can write the *new profit function* $P_i(p_i(\alpha_i, \alpha_j), p_j(\alpha_i, \alpha_j))$ (hereafter P_i) of each firm as:

$$P_i = (1 - \alpha_i)\Pi_i + \frac{\alpha_i}{\alpha_i + \alpha_j} f(\alpha_1\Pi_1, \alpha_2\Pi_2) = (1 - \alpha_i)\Pi_i + \frac{\alpha_i}{\alpha_i + \alpha_j} \gamma (\alpha_i\Pi_i + \alpha_j\Pi_j)$$

or

$$P_i = [1 - (1 - C_i\gamma)\alpha_i] \Pi_i + C_i\gamma\alpha_j\Pi_j$$

We consider a two-stage game whose sequences are thus defined. In the first stage of the game, firms choose the optimal α_i to invest. In the second stage of the game, firms select p_i .

In the *first stage of the game*, for α_1 and α_2 firms simultaneously solve:

$$\begin{aligned} \text{Max}_{\alpha_1} \quad P_1 &= [1 - (1 - \gamma C_1)\alpha_1] \Pi_1 + \gamma C_1 \alpha_2 \Pi_2 \\ \text{Max}_{\alpha_2} \quad P_2 &= [1 - (1 - \gamma C_2)\alpha_2] \Pi_2 + \gamma C_2 \alpha_1 \Pi_1 \end{aligned}$$

In the *second stage of game*, for p_1 and p_2 firms simultaneously solve:

$$\begin{aligned} \text{Max}_{\alpha_1} \quad P_1 &= [1 - (1 - \gamma C_1)\alpha_1] \Pi_1 + \gamma C_1 \alpha_2 \Pi_2 \\ \text{Max}_{\alpha_2} \quad P_2 &= [1 - (1 - \gamma C_2)\alpha_2] \Pi_2 + \gamma C_2 \alpha_1 \Pi_1 \end{aligned}$$

3.1 Solving the second-stage of the game

To find the subgame perfect Nash equilibrium (SPNE), we begin by solving subgames in the second-stage. Recall that, in the second stage, firms are looking for prices that maximize their profits.

Proposition 5 *If $(1 - \gamma C_1)\alpha_1 + \gamma C_1 \alpha_2 = 1$ and $(1 - \gamma C_2)\alpha_2 + \gamma C_2 \alpha_1 = 1$, then any prices (p_1, p_2) such that $c \leq p_1 = p_2 \leq p_m$ are NE_a in the second stage of the game*

Proof. (p_1, p_2) such that $c \leq p_1 = p_2 \leq p_m$ are NE_a if and only if no firm wants to deviate from those prices by fixing a price p'_i above or below. In fact:

$$c \leq p_1 = p_2 = p \leq p_m \Rightarrow \Pi_1 = \Pi_2 \geq 0$$

$$\Pi_1 = \frac{1}{2} (p_1 - c) (1 - p_1) = \frac{1}{2} (p - c) (1 - p) = \Pi_2$$

$$P_1 = [1 - (1 - \gamma C_1)\alpha_1 + \gamma C_1\alpha_2] \Pi_1$$

$$P_1 = \frac{1}{2} [1 - \alpha_1 + \gamma\alpha_1] (p - c) (1 - p)$$

$$P_2 = \frac{1}{2} [1 - \alpha_2 + \gamma\alpha_2] (p - c) (1 - p)$$

Suppose that:

$$i) p_1 = p_2 - \varepsilon \ (\varepsilon > 0) \iff \Pi_1 = (1 - p_1) (p_1 - c) \text{ and } \Pi_2 = 0$$

$$P'_1 = [1 - (1 - \gamma C_1)\alpha_1] \Pi_1 = [1 - (1 - \gamma C_1)\alpha_1] (1 - p_1) (p_1 - c)$$

If $p_1 \leq p_m$ (monopolistic price), then $p_1 = p - \varepsilon$.

$$\text{For } \varepsilon \text{ very small}^7, P'_1 \simeq [1 - (1 - \gamma C_1)\alpha_1] (1 - p) (p - c) \leq P_1 \Leftrightarrow$$

$$[1 - (1 - \gamma C_1)\alpha_1] \leq \frac{1}{2} [1 - \alpha_1 + \gamma\alpha_1] \quad (9)$$

$$ii) p_1 = p_2 + \varepsilon \ (\varepsilon > 0) \iff \Pi_2 = (1 - p_2) (p_2 - c) > 0 \text{ and } \Pi_1 = 0$$

$$P''_1 = \gamma C_1\alpha_2 \Pi_2 = \gamma C_1\alpha_2 (1 - p_2) (p_2 - c) \leq P_1 \Leftrightarrow$$

$$\gamma C_1\alpha_2 \leq \frac{1}{2} [1 - \alpha_1 + \gamma\alpha_1] \quad (10)$$

Equations (9) and (10) represent the non-deviation conditions for firms 1 and are both satisfied when $[1 - (1 - \gamma C_1)\alpha_1] = \gamma C_1\alpha_2$ or $(1 - \gamma C_1)\alpha_1 + \gamma C_1\alpha_2 = 1$.

Likewise, we can show that firm 2 will not deviate if $[1 - (1 - \gamma C_2)\alpha_2] = \gamma C_2\alpha_1$ or $(1 - \gamma C_2)\alpha_2 + \gamma C_2\alpha_1 = 1$.

Conclusion: if $(1 - \gamma C_1)\alpha_1 + \gamma C_1\alpha_2 = 1$ & $(1 - \gamma C_2)\alpha_2 + \gamma C_2\alpha_1 = 1$, then any prices (p_1, p_2) such that $c \leq p_1 = p_2 \leq p_m$ are NE_a in the second-stage of the game. ■

⁷There is no reason for not to suppose that ε is very small. For instance, firms need to decrease or increase just slightly to get or to lose the entire market.

Proposition 6 *If $(1 - \gamma C_1)\alpha_1 + \gamma C_1\alpha_2 = 1$ or $(1 - \gamma C_2)\alpha_2 + \gamma C_2\alpha_1 = 1$, then any prices (p_i, p_j) such that $c \leq p_i = p_m < p_j$ are NE_a in the second stage of the game*

Proof. (p_1, p_2) s. t. $c \leq p_2 = p_m < p_1$ are NE_a if and only if no firm has interest to deviate from those prices by fixing a price p'_i above or below.

$$A.- c \leq p_2 = p_m < p_1 \Rightarrow \Pi_1 = 0 \text{ and } \Pi_2 = (p_2 - c)(1 - p_2) > 0$$

$$P_1 = \gamma C_1\alpha_2\Pi_2 = \gamma C_1\alpha_2(p_2 - c)(1 - p_2)$$

$$P_2 = [1 - (1 - \gamma C_2)\alpha_2]\Pi_2 = [1 - (1 - \gamma C_2)\alpha_2](p_2 - c)(1 - p_2)$$

Suppose that:

$$i) p_1 = p_2 - \varepsilon \ (\varepsilon > 0) \iff \Pi_1 = (1 - p_1)(p_1 - c) \text{ and } \Pi_2 = 0$$

$$P'_1 = [1 - (1 - \gamma C_1)\alpha_1]\Pi_1 = [1 - (1 - \gamma C_1)\alpha_1](1 - p_2 + \varepsilon)(p_2 - \varepsilon - c)$$

$$\text{For } \varepsilon \text{ very small, } P'_1 \simeq [1 - (1 - \gamma C_1)\alpha_1](1 - p_2)(p_2 - c) < P_1 \iff$$

$$[1 - (1 - \gamma C_1)\alpha_1] < \gamma C_1\alpha_2 \quad (11)$$

Equation (11) represent the non-deviation conditions for firm 1 and can be rewritten as: $(1 - \gamma C_1)\alpha_1 + \gamma C_1\alpha_2 > 1$

$$B.- c \leq p_1 = p_m < p_2 \Rightarrow \Pi_1 = (p_1 - c)(1 - p_1) \text{ and } \Pi_2 = 0$$

$$P_1 = [1 - (1 - \gamma C_1)\alpha_1]\Pi_1 = [1 - (1 - \gamma C_1)\alpha_1](p_1 - c)(1 - p_1)$$

$$P_2 = \gamma C_2\alpha_1\Pi_1 = \gamma C_2\alpha_1(p_1 - c)(1 - p_1)$$

Suppose that:

$$i) p_2 = p_1 - \varepsilon \ (\varepsilon > 0) \iff \Pi_2 = (1 - p_2)(p_2 - c) \text{ and } \Pi_1 = 0$$

$$P'_2 = [1 - (1 - \gamma C_2)\alpha_2]\Pi_2 = [1 - (1 - \gamma C_2)\alpha_2](1 - p_1 + \varepsilon)(p_1 - \varepsilon - c)$$

$$\text{For } \varepsilon \text{ very small, } P'_2 \simeq [1 - (1 - \gamma C_2)\alpha_2](1 - p_1)(p_1 - c) < P_1 \iff$$

$$[1 - (1 - \gamma C_2)\alpha_2] < \gamma C_2\alpha_1 \quad (12)$$

Equation (12) represent the non-deviation conditions for firm 2 and can be rewritten as: $(1 - \gamma C_2)\alpha_2 + \frac{1}{2}\gamma\alpha_1 > 1$

Conclusion: if $(1 - \gamma C_2)\alpha_2 + \gamma C_2\alpha_1 > 1$ or $(1 - \gamma C_1)\alpha_1 + \gamma C_1\alpha_2 > 1$, then any prices (p_i, p_j) such that $c \leq p_i = p_m < p_j$ constitute NE_a in the second-stage of the game. ■

Proposition 7 *If $(1 - \gamma C_2)\alpha_2 + \gamma C_2\alpha_1 < 1$ or $(1 - \gamma C_1)\alpha_1 + \gamma C_1\alpha_2 < 1$, then any price (p_1, p_2) such that $p_1 = p_2 = c$ is NE in the second stage of the game*

Proof. (p_1, p_2) s.t. $p_1 = p_2 = c$ is NE if and only if no firm has interest to deviate from those prices to fix a price p'_i above or below.

$$p_2 = p_2 = c \Rightarrow \Pi_1 = 0 \text{ and } \Pi_2 = 0$$

$$P_1 = [1 - (1 - \gamma C_1)\alpha_1] \Pi_1 + \gamma C_1\alpha_2\Pi_2 = 0$$

$$P_2 = [1 - (1 - \gamma C_2)\alpha_2] \Pi_2 + \gamma C_2\alpha_1\Pi_1 = 0$$

We shall study separately the deviation for both firms. Let us check first for firm 1. Suppose that:

$$i) p_1 = p_2 - \varepsilon \text{ (} p_1 < p_2 \text{ and } p_1 < c) \Rightarrow \Pi_1 < 0 \text{ and } \Pi_2 = 0$$

$$P'_1 = [1 - (1 - \gamma C_1)\alpha_1] \Pi_1 < 0$$

$$\Rightarrow P'_1 < P_1 = 0 \Rightarrow \text{Firm 1 has no interest by fixing a price below } p_2$$

$$ii) p_1 = p_2 + \varepsilon \text{ (} p_1 > p_2 = c) \iff \Pi_2 = (1 - p_2)(p_2 - c) = 0 \text{ and } \Pi_1 = 0 \text{ (firm 1 does not produce)}$$

$$P''_1 = \gamma C_1\alpha_2\Pi_2 = 0$$

$$P''_1 = 0 = P_1 \Rightarrow \text{Firm 1 has no interest by fixing a price above } p_2$$

Let us check now for firm 2. Suppose that:

$$i) p_2 = p_1 - \varepsilon \text{ (} p_2 < p_1 \text{ and } p_2 < c) \Rightarrow \Pi_2 < 0 \text{ and } \Pi_1 = 0 \text{ (firm 1 does not produce)}$$

$$P'_2 = [1 - (1 - \gamma C_2)\alpha_2] \Pi_2 < 0$$

$$\Rightarrow P'_2 < P_2 = 0 \Rightarrow \text{Firm 2 has no interest by fixing a price below } p_1$$

$$ii) p_2 = p_1 + \varepsilon \text{ (} p_2 > p_1 = c) \iff \Pi_1 = (1 - p_1)(p_1 - c) = 0 \text{ and } \Pi_2 = 0 \text{ (firm 2 does not produce)}$$

$$P''_2 = \gamma C_2\alpha_1\Pi_1 = 0$$

$$P''_2 = 0 = P_2 \Rightarrow \text{Firm 2 has no interest by fixing a price above } p_1$$

Conclusion: If $(1 - \gamma C_2)\alpha_2 + \gamma C_2\alpha_1 < 1$ or $(1 - \gamma C_1)\alpha_1 + \gamma C_1\alpha_2 < 1$, then any price, (p_1, p_2) s.t. $p_1 = p_2 = c$ is a NE in the second-stage of the game. ■

The second-stage being entirely solved and NE_a being found, we can thus move to the first-stage of the game in order to find $SPNE_a$

3.2 Solving the first-stage of the game

In the first-stage of the game, firms choose the α_i optimal maximizing their profit to share with their rival.

Solving backwards, we have solved the second-stage of the game in the previous section and have found NE_a in prices summarized below⁸:

i) $(p_1, p_2) / p_1 = p_2 = c$ if $(1 - \gamma C_2)\alpha_2 + \gamma C_2\alpha_1 < 1$ or $(1 - \gamma C_1)\alpha_1 + \gamma C_1\alpha_2 < 1$ with:

$$\begin{cases} P_1 = 0 \\ P_2 = 0 \end{cases}$$

ii) $(p_1, p_2) / c \leq p_1 = p_2 \leq p_m$ if $(1 - \gamma C_2)\alpha_2 + \gamma C_2\alpha_1 = 1$ & $(1 - \gamma C_1)\alpha_1 + \gamma C_1\alpha_2 = 1$ with:

$$\begin{cases} P_1 = 2\gamma\alpha_2(p - c)(1 - p) \\ P_2 = 2\gamma\alpha_1(p - c)(1 - p) \end{cases}$$

iii) $(p_1, p_2) / c \leq p_i = p_m < p_j$ if $(1 - \gamma C_2)\alpha_2 + \gamma C_2\alpha_1 > 1$ or $(1 - \gamma C_1)\alpha_1 + \gamma C_1\alpha_2 > 1$ with:

$$\begin{cases} P_j = C_j\gamma\alpha_i(p_m - c)(1 - p_m) \\ P_i = [1 - (1 - C_i\gamma)\alpha_i](p_m - c)(1 - p_m) \end{cases}$$

Now, in the current section, we draw our attention to the first-stage of the game searching for $SPNE_a$ in α_i .

Proposition 8 *The strategies $(\alpha_1, p_1(\alpha_1, \alpha_2))$, $(\alpha_2, p_2(\alpha_1, \alpha_2))$ s.t.:*

$$\begin{aligned} & i) \alpha_i \in]0, 1[\text{ \& } (1 - \gamma C_2)\alpha_2 + \gamma C_2\alpha_1 = 1 \text{ \& } (1 - \gamma C_1)\alpha_1 + \gamma C_1\alpha_2 = 1 \\ & ii) \begin{cases} p_1^* = p_2^* = c \text{ if } (1 - \gamma C_2)\alpha_2 + \gamma C_2\alpha_1 < 1 \text{ or } (1 - \gamma C_1)\alpha_1 + \gamma C_1\alpha_2 < 1 \\ p_1^* = p_2^* = p_m \text{ if } (1 - \gamma C_2)\alpha_2 + \gamma C_2\alpha_1 = 1 \text{ \& } (1 - \gamma C_1)\alpha_1 + \gamma C_1\alpha_2 < 1 \\ c \leq p_i = p_m < p_j \text{ if } (1 - \gamma C_2)\alpha_2 + \gamma C_2\alpha_1 > 1 \text{ or } (1 - \gamma C_1)\alpha_1 + \gamma C_1\alpha_2 > 1 \end{cases} \end{aligned}$$

⁸One can easily check that: if $(1 - C_1\gamma)\alpha_1 + C_1\gamma\alpha_2 = 1$ (resp. $(1 - C_2\gamma)\alpha_2 + C_2\gamma\alpha_1 = 1$) then $\frac{1}{2}[1 - (1 - C_1\gamma)\alpha_1 + C_1\gamma\alpha_2] = 2\gamma\alpha_2$ (resp. $\frac{1}{2}[1 - (1 - C_2\gamma)\alpha_2 + C_2\gamma\alpha_1] = 2\gamma\alpha_1$).

are $SPNE_a$ of the game.

Proof. The strategies $(\alpha_1, p_1(\alpha_1, \alpha_2))$, $(\alpha_1, p_1(\alpha_1, \alpha_2))$ s.t. *i*) and *ii*) are satisfied, are $SPNE_a$ of the game if and only if no firm has interest to deviate from those prices by choosing a α'_i above or below. Because of the multiplicity of α_i , we investigate separately the deviation for each firm.

Let us check first for firm 1. Suppose that:

$$i) \alpha'_1 < \alpha_1 \Rightarrow (1 - \gamma C_1)\alpha'_1 + \gamma C_1\alpha_2 < 1 \Rightarrow$$

$$P'_1 = 0 < P_1 = 2\gamma\alpha_2(p_m - c)(1 - p_m) \quad (13)$$

$$ii) \alpha'_1 > \alpha_1 \Rightarrow (1 - \gamma C_1)\alpha'_1 + \gamma C_1\alpha_2 > 1 \Rightarrow$$

$$P''_1 = \gamma C_1\alpha_2(p_m - c)(1 - p_m) < P_1 = 2\gamma\alpha_2(p_m - c)(1 - p_m) \quad (14)$$

(13) and (14) show that firm 1 has no interest to deviate.

Now, let us check for firm 2. Suppose that:

$$i) \alpha'_2 < \alpha_2 \Rightarrow (1 - \gamma C_2)\alpha'_2 + \gamma C_2\alpha_1 < 1 \Rightarrow$$

$$P'_2 = 0 < P_2 = 2\gamma\alpha_1(p_m - c)(1 - p_m) \quad (15)$$

$$ii) \alpha'_2 > \alpha_2 \Rightarrow (1 - \gamma C_2)\alpha'_2 + \gamma C_2\alpha_1 > 1 \Rightarrow$$

$$P''_2 = [1 - (1 - \gamma C_2)\alpha'_2](p_m - c)(1 - p_m) < P_2 = 2\gamma\alpha_1(p_m - c)(1 - p_m) \quad (16)$$

(15) and (16) show that firm 2 has no interest to deviate.

Conclusion: The strategies $(\alpha_1, p_1(\alpha_1, \alpha_2))$, $(\alpha_1, p_1(\alpha_1, \alpha_2))$ s.t. *i*) and *ii*) are satisfied, are $SPNE_a$ of the game. ■

4 Conclusion

This paper has shed light on how two firms in a duopoly market may successfully form a joint venture. Furthermore and more importantly, it brings to light that *a joint venture might be used to conceal the profit-sharing* (maybe forbidden) *strategy*.

Several extensions are readily suggested. The first and natural one is the extension of our analysis to more than two firms. The second one is to consider different marginal costs. Finally, the third one is to move from the homogeneous market to an heterogeneous market. Such extensions should be straightforward. However, we leave them for future reasearch

5 References

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